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# Choosing Line Lengths for Calibrating Network Analyzers

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*Abstract* —**Equations, examples, and a table are given to help choose the best length for a precision transmission line which is used in some methods for calibrating a network analyzer. One line will cover a frequency range of about 10:1. Two lines will cover a range of about 65:1.**

## I. Introduction

Some techniques for calibrating a network analyzer use a length of precision transmission line as the standard. Examples of such techniques are the "thru-reflect-line" technique [1] and the "thru-short-delay" technique [2]–[5]. One problem with using a length of line as the standard is that its electrical length must not be too near multiples of 180°, or the solution for the constants characterizing the network analyzer becomes ill-conditioned. An electrical length of $90° + 180°m$ is ideal, where $m = 0, 1, 2 \cdots$. Another problem is that the physical length of a line whose electrical length is less than 180° may be too short to be practical. Three methods of choosing the line lengths to avoid these problems are given in this short paper.

## II. Short Lines

Let the frequency range of the network analyzer be from $f_1$ to $f_2$. If $f_2/f_1$ is less than about 10, one line will frequently provide satisfactory performance over the whole operating range if its electrical length is 180° at the frequency $f_1 + f_2$, as shown in Fig. 1. If the effective phase shift through the line is defined as the absolute difference between the actual phase shift and the closer of 0° or 180°, then the minimum effective phase shift through the line will be the same at both $f_1$ and $f_2$, and the phase will be 90° at the center of the band.

The phase shift through an air-dielectric coaxial transmission line is [6]

$$\phi = 12 fl \text{ deg} \tag{1}$$

where $f$ is the frequency in gigahertz and $l$ is the length in centimeters. For the phase shift to be 180° at $f_1 + f_2$, the length of the coaxial line must be

$$l = \frac{15}{f_1 + f_2} \text{ cm}, \qquad \text{for } f \text{ in gigahertz.} \tag{2}$$

Using (2) in (1), the phase shift at $f_1$ is

$$\phi_{f_1} = \frac{180}{1 + f_2/f_1} \text{ deg.} \tag{3}$$

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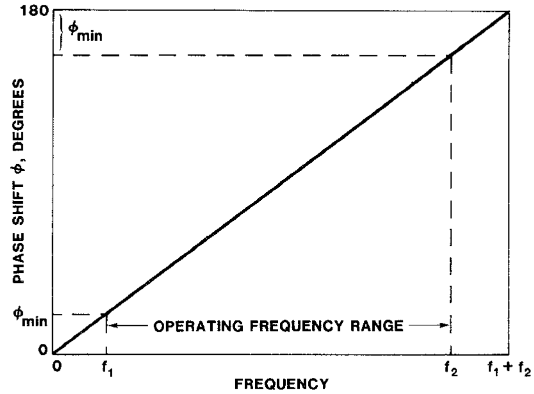


Fig 1 Phase shift through a single transmission line designed to cover the frequency range $f_1$ to $f_2$. The optimum length is that which gives a phase shift of 180° at $f_1 + f_2$.

This is also the effective phase shift at $f_2$. If the effective phase shift at $f_1$ and $f_2$ is too small, the frequency range $f_1$ to $f_2$ may be broken into two ranges, $f_1$ to $f_i$ and $f_i$ to $f_2$, where $f_i$ is some intermediate frequency. A practical choice for $f_i$ is such that $f_2/f_i = f_i/f_1 = \sqrt{f_2/f_1}$. If the line lengths for each range are chosen as described above, the phase angles at $f_1$, $f_i$, and $f_2$ calculated from (3) for the smaller ranges will then all be equal and as large as possible. This idea can obviously be extended to more than two ranges.

At the higher frequencies, the physical length $l$ calculated from (2) may be too small to be practical. Then one must use either one or two longer lines as discussed below.

## III. One Long Line

As an alternative to the above, it is possible to use a longer line over an arbitrarily wide frequency range, although operation at certain bands of frequencies must be excluded to avoid associated phase shifts too near multiples of 180°. In this section is derived an expression for choosing equally spaced frequencies having phase angles which are a specified minimum distance from multiples of 180°.

Let the calibration frequencies be equally spaced a distance $\Delta f$ apart. The corresponding changes in phase, $\Delta\phi$, from (1) will be

$$\Delta\phi = 12 l \Delta f. \tag{4}$$

If the line length or the frequencies are chosen as shown in Fig. 2(a) so that the phase shifts nearest to 0° and 180° are a distance

$$\phi_{min} = \frac{\Delta\phi}{2} \tag{5}$$

from 0° and 180°, then the effective phase shifts at all frequencies of measurement will be equal to or greater than $\phi_{min}$. Adding the phase shifts from 0° to 180° in Fig. 2(a) gives

$$180° = 2\phi_{min} + n\,\Delta\phi, \qquad n = 0, 1, 2, \cdots \tag{6}$$

where $n = 4$ in Fig. 2(a). Eliminating $\Delta\phi$ from (5) and (6) leads to

$$\phi_{min} = \frac{90}{1 + n}. \tag{7}$$

Thus $\phi_{min}$ is restricted to values of 90°, 45°, 22.5°, 18°, etc. Eliminating $\Delta\phi$ from (4) and (5) gives

$$\Delta f = \frac{\phi_{min}}{6l}. \tag{8}$$

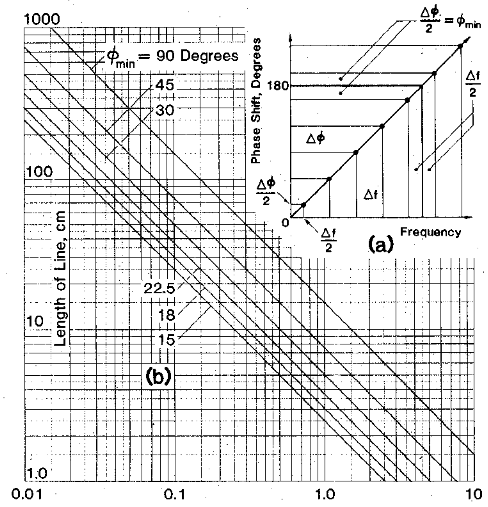


Fig. 2. (a) Choosing equally spaced frequencies when using a long line to avoid phase shifts near 180°. (b) The minimum effective phase shift $\phi_{min}$ for a line of length $l$ when the frequencies of measurement are equally spaced a distance $\Delta f$ apart, assuming that the first frequency is given by (9).

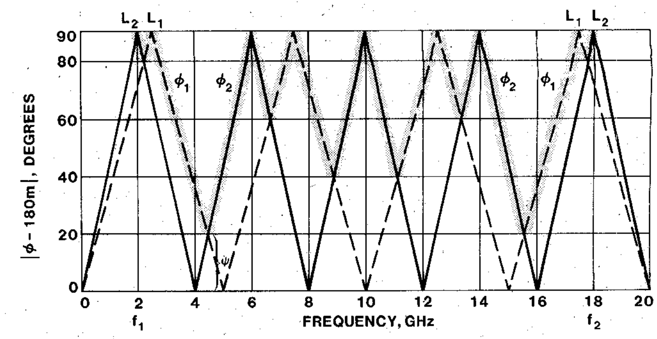


Fig. 3. Effective phase shift through two lines, $l_1 = 4l$ and $l_2 = 5l$, whose difference in length is $l$, the optimum length of a single line given by (2). For the operating frequency range of 2 to 18 GHz in this example, two longer lines of lengths 3 and 3.75 cm are more practical than one short line of length $l = 0.75$ cm.

This equation is plotted in Fig. 2(b) for values of $\phi_{min} = 90°$ to 15° obtained from (7) when $n = 0$ to 5. For a given length of line, this set of curves indicates the frequency increments that will have corresponding phase shifts no closer to any multiple of 180° than $\phi_{min}$. This assumes that the first frequency $f_1$ is one of the points in Fig. 2(a), namely,

$$f_1 = \frac{\Delta f}{2} + n_1 \Delta f, \qquad n_1 = 0, 1, 2, \cdots. \tag{9}$$

The individual frequencies must satisfy (8) and (9), and therefore cannot be chosen arbitrarily.

For example, suppose one has a 30-cm air line. Fig. 2(b) indicates that this line could be used in the calibration of an ANA at frequencies separated 0.1 GHz apart with a minimum effective phase shift of 18°. The first frequency might be 0.75 GHz ($0.1/2 + 7 * 0.1$), obtained from (9).

The difference between frequencies can also be any multiple of $\Delta f$ obtained from Fig. 2(b). For the 30-cm air line, $\Delta f$ could be 0.1, 0.2, 0.3, etc., all with $\phi_{min} = 18°$.

In principle, by making the line sufficiently long, the frequency interval $\Delta f$ can be made arbitrarily small, and thus "continuous" frequency coverage can be approached. In practice, however, the line must be limited to some practical length. The idea of using two long lines of different lengths will be explored next.

## IV. Two Long Lines

As is evident from the above, one long line does provide for continuous coverage except for discrete bands around those frequencies yielding phase shifts which are multiples of 180°. The next idea is to choose two lines of different lengths such that, insofar as this is convenient or possible, these bands do not overlap.

One method which has been found useful in this context is to choose two lines of length $l_1$ and $l_2$ such that their lengths are multiples of $l$ as defined by (2) and also differ by $l$. That is, let

$$l_1 = nl \tag{10}$$

$$l_2 = (n+1)l \tag{11}$$

where $n$ is an integer yet to be determined. If $\phi_1$ and $\phi_2$ are the corresponding phase shifts through these two lines, the difference in phase becomes

$$\phi_2 - \phi_1 = 180 \frac{f}{f_1 + f_2} \tag{12}$$

which is obtained from (10), (11), and (2) in (1). At frequencies less than $f_1 + f_2$, the phase difference will evidently be less than 180° so that if one line has a phase shift which is a multiple of 180°, the phase shift provided by the other cannot also be a multiple of 180°. There will always be one line with a reasonably large effective phase shift, defined here as the absolute difference between $\phi$ and the nearest multiple of 180°.

The effective phase shift for an example with $n = 4$ is shown in Fig. 3 where $f_1$ to $f_2$ is 2 to 18 GHz. The lines cannot be used at frequencies where these curves approach the 0° line, so different lines must be used at different frequencies. The shaded upper outline of the curves indicates which line is preferred at a given frequency to keep the effective phase shift large. The computer controlling the ANA could select all frequencies appropriate for a given line so that each line is inserted only once during the calibration.

As one can see from Fig. 3, the minimum effective phase shift when using two lines can occur at either $f_1$ (and $f_2$) or at that frequency where one must change from one line to the other to avoid phase angles too close to 180°. This latter phase angle is labeled $\psi$ in Fig. 3. A general expression for $\psi$ can be obtained as follows. Note that $\psi$ occurs at that frequency in Fig. 3 where the phase shift through $l_1$ is $180 - \psi$, while for the second line the phase shift has already gone through 180° and is $180 + \psi$. By use of (1) this evidently occurs at a frequency $f$ given by

$$f = \frac{180 - \psi}{12 l_1} = \frac{180 + \psi}{12 l_2}. \tag{13}$$

With the help of (10) and (11), this yields $\psi$ as

$$\psi = \frac{180}{2n+1}. \tag{14}$$

Values of $\psi$ calculated from (14) are listed at the top of Table I for different values of $n$. The rest of the table gives the maximum of either $\phi_{1f1}$ or $\phi_{2f1}$, where $\phi_{1f1}$ is the phase shift of line 1 at $f_1$, and $\phi_{2f1}$ is the corresponding phase shift of line 2 at $f_1$. These phase angles at $f_1$ are obtained from (1) using $l_1$ and $l_2$ in place of $l$. In general, the effective phase shift through one line or the other can be kept greater than the minimum of $\psi$ or $\max(\phi_{1f1}, \phi_{2f1})$. Below the diagonal line in Table I, $\phi_{2f1}$ is greater than $\phi_{1f1}$ so $\phi_{2f1}$ is listed. The stair-step line indicates those

TABLE I

PHASE ANGLES $\psi$ AND THE MAXIMUM OF $\phi_1$ AND $\phi_2$ AT $f_1$ AS A FUNCTION OF $n$ AND $f_2/f_1$, THE RATIO OF STOP-TO-START FREQUENCY. THE MINIMUM EFFECTIVE PHASE SHIFT WHEN USING TWO LINES IS THE SMALLER OF $\psi$ OR $\max(\phi_{1/f1}, \phi_{2f1})$. THE STAIR-STEP LINE INDICATES THE VALUES OF $n$ AND $f_2/f_1$ WHERE $\psi = \max(\phi_{1/f1}, \phi_{2f1})$.

| n = | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| ψ = | | 60 | 36 | 26 | 20 | 16 | 14 | 12 |
| $f_2/f_1$ | Max($\phi_{1f1}$, $\phi_{2f1}$) | | | | | | | |
| 1 | 90 | 90 | 90 | 90 | 90 | 90 | 90 | 90 |
| 2 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 3 | 45 | 90 | 90 | 45 | 45 | 90 | 90 | 45 |
| 4 | 36 | 72 | 72 | 72 | 36 | 36 | 72 | 72 |
| 5 | 30 | 60 | 90 | 90 | 60 | 30 | 30 | 60 |
| 6 | 26 | 51 | 77 | 77 | 77 | 51 | 26 | 26 |
| 7 | 23 | 45 | 68 | 90 | 90 | 68 | 45 | 23 |
| 8 | 20 | 40 | 60 | 80 | 80 | 80 | 60 | 40 |
| 9 | 18 | 36 | 54 | 72 | 90 | 90 | 72 | 54 |
| 10 | 16 | 33 | 49 | 65 | 82 | 82 | 72 | 65 |
| 11 | 15 | 30 | 45 | 60 | 75 | 90 | 90 | 75 |
| 12 | 14 | 28 | 42 | 55 | 69 | 83 | 83 | 83 |
| 13 | 13 | 26 | 39 | 51 | 64 | 77 | 90 | 90 |
| 14 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 84 |
| 15 | 11 | 23 | 34 | 45 | 56 | 68 | 79 | 90 |
| 16 | 11 | 21 | 32 | 42 | 53 | 64 | 74 | 85 |
| 17 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| 18 | 9 | 19 | 28 | 38 | 47 | 57 | 66 | 76 |
| 19 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 |
| 20 | 9 | 17 | 26 | 34 | 43 | 51 | 60 | 69 |
| 22 | 8 | 16 | 23 | 31 | 39 | 47 | 55 | 63 |
| 24 | 7 | 14 | 22 | 29 | 36 | 43 | 50 | 58 |
| 26 | 7 | 13 | 20 | 27 | 33 | 40 | 47 | 53 |
| 28 | 6 | 12 | 19 | 25 | 31 | 37 | 43 | 50 |
| 30 | 6 | 12 | 17 | 23 | 29 | 35 | 41 | 46 |
| 32 | 5 | 11 | 16 | 22 | 27 | 33 | 38 | 44 |
| 34 | 5 | 10 | 15 | 21 | 26 | 31 | 36 | 41 |
| 36 | 5 | 10 | 15 | 19 | 24 | 29 | 34 | 39 |
| 38 | 5 | 9 | 14 | 18 | 23 | 28 | 32 | 37 |
| 40 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 |
| 42 | 4 | 8 | 13 | 17 | 21 | 25 | 29 | 33 |
| 44 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
| 46 | 4 | 8 | 11 | 15 | 19 | 23 | 27 | 31 |
| 48 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 29 |
| 50 | 4 | 7 | 11 | 14 | 18 | 21 | 25 | 28 |
| 55 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 |
| 60 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |
| 65 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 |
| 70 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 |
| 75 | 2 | 5 | 7 | 9 | 12 | 14 | 17 | 19 |
| 80 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 |
| 85 | 2 | 4 | 6 | 9 | 10 | 13 | 15 | 17 |
| 90 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| 95 | 2 | 4 | 6 | 8 | 9 | 11 | 13 | 15 |
| 100 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 |

values of $f_2/f_1$ and $n$ where $\psi = \phi_{2f1}$. For example, when $f_2/f_1 = 65$ and $n = 5$, $\psi = \phi_{2f1} = 16°$. Any other value of $n$ would give either $\psi$ or $\phi_{2f1}$ smaller than 16°, indicating that $n = 5$ is the best choice at $f_2/f_1 = 65$. Experience on a dual six-port ANA at NBS indicates that it is probably not practical to let the minimum effective phase shift through the line fall below about 16° when calibrating the network analyzer.

The $n = 0$ column in Table I can be used to find the value of $\phi$ at $f_1$ when using a single line.

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# Optical Injection Locking of BARITT Oscillators

R. HEIDEMANN AND D. JÄGER

*Abstract* — **Optical injection locking of BARITT oscillators is investigated. Preliminary experimental results are presented for the first time. A simple first-order locking theory gives reasonable agreement with measurements.**

In modern high-performance radar systems, it is advantageous to use many low-power transmitters in an electronically steerable phase-array configuration. All these low-power transmitters have to oscillate with the same frequency and a given phase relation using locking signals distributed to the individual oscillators. This may be achieved by optical injection locking if electrooptic microwave semiconductor devices—e.g., IMPATT's—are used. These optical locking signals can then be distributed by optical fibers with the advantage of low loss, negligible dispersion, and low weight as compared with conventional microwave transmission lines.

Forrest and Seeds [1] have shown that optical injection locking of IMPATT oscillators should be possible. A large-signal theory of an IMPATT diode under the influence of intensity modulated light has been developed and a locking bandwidth of about 0.5 percent at *X*-band has been predicted but experimental results are still lacking. Optical injection locking of bipolar transistor oscillators at 1.8 GHz has been realized by Yen and Barnoski [2]. Recently, Sallas and Forrest [3] have demonstrated optical injection locking of GaAs MESFET oscillators at 2.35 GHz. With an optical power of about 1 mW, a locking bandwidth of 0.2 percent has been achieved.

In the following, the optical locking behavior of UHF–MSM–BARITT oscillators is investigated. To determine the locking bandwidth, the simple lumped model of BARITT oscillators as shown in Fig. 1 with the small-signal admittance $Y_D$ of the BARITT device, the load admittance $Y_L$, and the ac locking current source $I_{Ph}$ is used. The hole- or electron-locking current $I_{Ph}$ is generated by illuminating one of the Schottky contacts with intensity modulated laser light (see inset in Fig. 2). Using the Adler criterion [4], one obtains a linear dependence of $\Delta\omega$ on $I_{Ph}$ according to

$$\Delta\omega = \frac{\omega_o}{Q_L} \frac{|I_{Ph}|}{(8G_L P_{HF})^{1/2}} \tag{1}$$

where $\omega_o$ is the oscillator frequency, $Q_L$ the loaded $Q$ factor of the oscillator, $G_L$ the load admittance, and $P_{HF}$ the microwave output power of the oscillator. Assuming optical generation of carriers within surface layers only, it is easy to show that the photocurrent is approximately given by [5]

$$|I_{Ph}| = \frac{q\eta P\lambda}{hc} \frac{\left[\sin^2(\omega_{Ph}\tau) + \{\cos(\omega_{Ph}\tau) - 1\}^2\right]^{1/2}}{\omega_{Ph}\tau} \tag{2}$$

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